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An explanation of the clock paradox

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Abstract. A simple explanation of the clock paradox of special relativity is obtained by considering the aberration of light.

Two observers O and O′ are stationary at a point. They have identical synchronized clocks. At time zero, O′ suddenly acquires a velocity v. Later O′ suddenly ceases to move and remains stationary at a point P at a distance L from O.

According to special relativity, a clock moving with velocity v records time at a smaller rate than a stationary clock. The ratio of the two rates is $\beta = \{1-(v^2/c^2)\}^{-1/2}$. Therefore, from the point of view of O, the clock of O′ records time less quickly while it is moving and it has a time lag of $(L/v)\{1-(1/\beta)\}$ when it becomes stationary.

As the observers separate, both clocks read zero. The observer O′ can regard himself as stationary. From his point of view, the clock of O records time more slowly than his own clock. As the point P approaches him, the clock of O lags behind his. Then his velocity changes and instantaneously the clock of O gains time in comparison with his clock. Consider another observer O_1 who was stationary at P throughout. He would see the clock of O steadily recording time and for him there would be no discontinuity. The light which would reach O_1 is the light which O′ sees as he moves towards P. Therefore, the observer O′ sees no discontinuity in the time shown by the clock of O.

There appears to be a contradiction which has misled some scientists and given difficulties to many students. Dingle (1956a, b) has argued that the clock of O cannot suddenly gain time and that, therefore, the theory of special relativity is incorrect. The explanation of how it gains time requires consideration of the details of what the observers see.

First consider the point of view of O. He observes O′ through telescopes and the directions of his telescopes have to be altered as O′ moves away from him. From these directions, the positions of events and the times of travel of light from these events can be calculated. The times of the events can be found by subtraction of the times of travel from the times of arrival of the light. At his own time $L\{(1/v)+(1/c)\}$, O observes O′ to stop at a distance L away with his clock reading $L/\beta v$. He calculates the time of this event to be L/v, so the clock of O′ lags behind his own clock by an amount $(L/v)\{1-(1/\beta)\}$. There is no discontinuity for O. He does not need suddenly to turn his telescope through a finite angle.

The observer O′ observes O through telescopes. Consider the positions of these telescopes in the space–time of O. They do not point towards him while they are moving because of aberration. With distances as shown in figure 1(*a*),

$$\frac{ha}{b} = x - \frac{vL}{c}.$$

When O′ stops, the positions of his telescopes have suddenly to be altered and afterwards they point at O as in figure 1(*b*). From the positions of the telescopes, O can deduce what O′ was observing. His deduction follows.

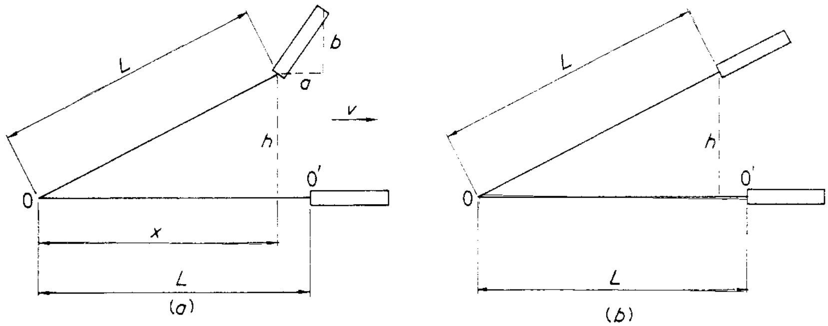

Figure 1. The positions of the telescopes of the observer O′ as seen by the observer O: (*a*) when O′ is about to stop, and (*b*) when O′ has just stopped.

Consider the point of view of O′. The positions of his telescopes when he is about to stop are shown in figure 2; distances parallel to $\boldsymbol{v}$ are multiplied by β. His time is $L/\beta v$ and he is seeing the clock of O showing a time of $(L/v)-(L/c)$. From the directions of his telescopes, he calculates that this light came from a point whose distance is

$$\frac{\beta ah}{b}+\beta(L-x) = \beta\left(x-\frac{vL}{c}+L-x\right) = \beta L\left(1-\frac{v}{c}\right).$$

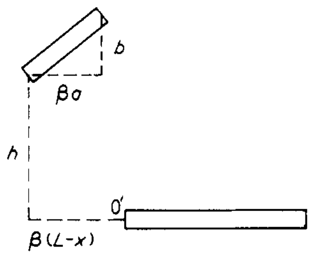

Figure 2. The position of the telescopes of O′ from his own point of view when he is about to stop.

He calculates the time of the emission of this light as

$$\frac{L}{\beta v}-\frac{\beta L}{c}\left(1-\frac{v}{c}\right) = \beta L\left(\frac{1}{v}-\frac{1}{c}\right).$$

Thus, for O′, the clock of O is recording time slowly. The ratio of the rates is β.

Then O′ suddenly has to alter the position of his telescopes. He observes the clock of O showing the same time $(L/v)-(L/c)$ but he finds that the clock is at a distance L from him. He calculates the time of emission of the light as

$$\frac{L}{\beta v}-\frac{L}{c},$$

and he finds that the clock of O is ahead of his own clock by

$$L\left(\frac{1}{v}-\frac{1}{c}\right)-L\left(\frac{1}{\beta v}-\frac{1}{c}\right)=\frac{L}{v}\left(1-\frac{1}{\beta}\right).$$

Thus the observers agree that the clock of O is ahead of that of O′ and agree about the amount that it is ahead.

The observers are not symmetric because the velocity of O′, with respect to an inertial frame, changes. The observer O′ sees no discontinuity in the time of the clock of O. When his velocity alters, there are discontinuities in the positions of his telescopes, the calculated distances of O from him and the calculated times of the travel of the light from the clock of O.

Aberration is a well established experimental fact. There can be no doubt that the clock of O would gain time for the observer O′ when his velocity changed. The above calculation shows that the gain of time exactly accounts for the discrepancy which Dingle pointed out. There is no paradox and the special theory of relativity does not lead to a contradiction.

In an inertial frame, time can be measured either with one clock and two telescopes or with a number of synchronized clocks. The two methods give identical results. If the velocity of a frame changes, synchronized clocks cannot be used, because clocks which were synchronized before the velocity altered would not be so afterwards. The necessity of using telescopes has not been appreciated and, in consequence, there are many conflicting views of the clock paradox, of which some are discussed by Rosser (1964).

To demonstrate the principles simply, an instantaneous change of velocity has been considered. Also the telescopes of O′ are in a special position. The light which leaves O at his time, $(L/v)-(L/c)$, reaches these telescopes simultaneously in the time of O. It has been assumed that these telescopes cease to move simultaneously in the time of O. If the telescopes were in different positions, the light might reach one before it stopped and the other after it stopped. For all positions, the observers would finally agree.

The general case of O′ being continuously accelerated with the direction of the acceleration changing is complicated. O′ could not directly determine the position of any event in the coordinates which move with him from light which reached him while he was being accelerated. If he tried to do so, he would get contradictory results from two pairs of telescopes. To find positions, he would need to know his accelerations in an inertial frame. He could then calculate his velocities in the inertial frame and correct his reading for aberration. In principle, he would find positions in the inertial frame and calculate positions in the accelerated frame.

The relation between the clock paradox and aberration has been shown. In general, there are relations between the Lorentz transformation and aberration. If special relativity is taught with consideration of telescopes and aberration, the subject becomes more concrete. Students have fewer difficulties because some parts of the theory, which otherwise are often puzzling, become easily understandable.

References

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